3   Give a sketch, for $0 \leqslant x \leqslant \pi/2$, of the curve

$$y = (\sin x - x \cos x),$$

and show that $0 \leqslant y \leqslant 1$.

Show that:

(i) $\displaystyle\int_0^{\pi/2} y\,dx = 2 - \frac{\pi}{2}$;

(ii) $\displaystyle\int_0^{\pi/2} y^2\,dx = \frac{\pi^3}{48} - \frac{\pi}{8}$.

Deduce that $\pi^3 + 18\pi < 96$.

Please provide answers to the following discussion questions. Don't include full calculations in your responses, just explore the question and try to anticipate routes through it.

1. How can you break down this curve into curves that are easier to think about? What checks can you do to make sure you are on the right track with your sketch?

2. What will be most tricky when integrating y in part (i)? What will you need to bring in from your mathematical toolkit?

3. How will you integrate y^2 in part (ii)? Which, if any, standard mathematical techniques will you use?

4. What sort of consideration might give rise to an inequality like the one at the end of this question? How might the (provided) answers to i) and ii) help you with this?

Submit your answers by e-mail to stepeasterschool@maths.org by Friday 16th March 2012 with the subject line: Lecture 3 Question 2